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Improving Pedagogy in the Developmental Mathematics Classroom

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Community colleges typically offer extensive developmental education programs to students with weak academic skills in order to prepare them for college-level coursework. Yet, for students referred to developmental mathematics education, rates of completion in developmental math courses and in college-level math courses required for a degree are particularly low. Failure to complete these courses prevents individuals from earning a college degree and is associated with a reduced likelihood of employment. This Brief summarizes a literature review that examines the research evidence concerning a potential means for improving course completion and learning outcomes among developmental math students: reforming mathematics classroom pedagogy. It concludes by offering recommendations for future research and for the adoption of particular instructional practices.

Typical developmental math pedagogy is thought to rely on procedural skill-building, which has been linked to better performance on standardized tests—but in order to understand mathematics, students need much more than procedural fluency (Hiebert & Grouws, 2007; Kilpatrick, Swafford, & Findell, 2001). Kilpatrick et al. (2001) identified five interdependent strands of mathematical learning that instructional practices must address to build mathematical proficiency: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. Research suggests that the forms of instruction discussed below may support additional components of mathematical learning beyond procedural fluency.

Examining Instructional Practices

Although the purpose of this review is to identify promising developmental math pedagogy, there is very little empirical research on this topic. Therefore, this Brief also reviews the literature on mathematics pedagogy in elementary and secondary schools and in college-level courses, focusing on empirical studies published within the last 20 years that evaluate the impact of an instructional practice on student outcomes. The studies are organized into six sets according to the main instructional approach focused on in the study. The sets are student collaboration, metacognition, problem representation, application, understanding student thinking, and computer-based learning. It is important to recognize that in the classroom, these approaches are not typically used in isolation but instead are often used in combination.

Student Collaboration

Overall, the literature on math pedagogy indicates that student collaboration has a positive impact on math learning. While most studies on this subject did not ensure treatment and control group equivalency, five rigorous elementary school student collaboration studies demonstrate that highly structured forms of student collaboration are especially effective for low-achieving math students. In addition, a randomized study from a developmental math classroom found support for an instructional approach in which students worked together to solve problems and received a group grade for their work (Dees, 1991).

With peer-assisted learning strategies (PALS) and peer-mediated instruction (PMI), students take turns acting as either the tutor who asks questions and provides feedback or the tutee who answers questions. Across the three studies of PALS and PMI (Fuchs, Fuchs, Phillips, Hamlett, & Karns, 1995; Fuchs et al., 1997; Fuchs, Fuchs, & Karns, 2001), results varied for students with different levels of math aptitude. Students with learning disabilities and average-achieving students usually made small gains in achievement compared to their counterparts in the control classrooms. Lower-achieving students made small to moderate gains. Higher-achieving students only benefited from paired work when assigned more complex math tasks. Similarly, a randomized study by Ginsburg-Block and Fantuzzo (1998) found that low-income, low-achieving third- and fourth-grade students who used a structured peer-tutoring format to review math skills outperformed comparable students who received traditional instruction. In contrast, Karper and Melnick's (1993) randomized study in a wealthy school district found that Team Accelerated Instruction had no impact on math achievement for elementary and middle school students. Overall, evidence suggests that structured student collaboration may be more beneficial for low-achieving students.

Metacognition

Instructional practices may promote metacognition through comprehension monitoring, cognitive strategy instruction, or using writing and questioning during the problem-solving process to foster self-reflection. A number of studies describe pedagogical practices designed to improve students' ability to monitor their problem-solving process, but only one employed a rigorous experimental design. In Tournaki's (2003) study, second-grade students with and without disabilities were randomized into a control group and two treatment groups, in which they received additional instruction in either strategy instruction (verbalizing problem-solving steps) or drill-and-practice strategies. Students with learning disabilities who received strategy instruction experienced large gains in performance on addition facts compared to similar students who received drill-and-practice instruction. There were no differences in addition facts achievement between students without

disabilities who received the strategy instruction and those who received drill-and-practice instruction. Tournaki concludes that explicit instruction in problem-solving strategies, even for tasks as simple as adding, may be especially important for students with learning disabilities.

Problem Representation

Five of the studies in this set provide compelling evidence that improving students' problem representation skills has a small to moderate positive effect on math learning. Four high-quality studies that took place in elementary and middle school classrooms (Brenner et al., 1997; Jitendra et al., 1998; Jitendra et al., 2009; Witzel, Mercer, & Miller, 2003) found support for the routine use of multiple representations during problem solving by teachers and students. In the only higher education study in this set, Chappell (2006) ensured that even though students self-selected into concept-based calculus and traditional calculus sections, the faculty and students across both groups were comparable. Unannounced classroom observations by faculty not directly involved in the study confirmed that in the concept-based sections, faculty taught students how to solve problems using numerical, graphical, and algebraic methods and connected new ideas to prior knowledge. In the control sections, faculty moved through the textbook teaching definitions and formulas in a linear manner. Students in the concept-based sections performed significantly better on midterm and final exams and were better able to transfer their understanding to unfamiliar concepts.

Application

Application-oriented instructional approaches teach math concepts through real-world problem solving. Research consistently finds a positive association between teaching math through application and improved performance on tests of conceptual understanding. However, in a rigorous study by Hickey, Moore, and Pellegrino (2001), the treatment effects observed were too small for the results to conclusively support the use of a video-based problem-solving series for elementary students. In many of the studies in this set, treatment students outperformed control students on tests of understanding, but there were no differences in performance on more traditional tests of procedural fluency (Bottge, Heinrichs, Chan, & Serlin, 2001; Bottge, Heinrichs, Mehta, & Hung, 2002; Hickey et al., 2001; Hollar & Norwood, 1999; O'Callaghan, 1998). While these practices show promise for improving students' conceptual understanding, they may require more time and preparation and sometimes do not improve students' computational skills (Brenner et al., 1997).

Understanding Student Thinking

The fifth set of practices encompasses instructional methods that help teachers understand student thinking and adjust their instruction to meet the needs of their students. These pedagogical practices include assessment methods that are used during instruction to monitor student progress and guide instruction, such as frequent testing, classroom assessment techniques, classroom voting, the Keystone Method, progress monitoring, and curriculum-based measurement.

While it is generally accepted that college instructors should adapt instruction to meet the needs of their students through meaningful, ongoing assessment, most studies targeting college and developmental education students did

not utilize comparison groups or compared non-equivalent groups, so their findings cannot confirm the positive impact of ongoing, formative assessment. The rigorous K-12 studies by Fuchs and Fuchs (1990), Fuchs, Fuchs, Hamlett, and Stecker (1991), and Fuchs, Fuchs, Hamlett, Phillips, and Bentz (1994) find strong support for using curriculum-based measurement that provides teachers with expert recommendations to make instructional changes with elementary school students with learning disabilities. However, given that all three studies were conducted by the same researchers in the same setting, confidence in the validity of these studies would be strengthened if their results were replicated elsewhere.

Computer-Based Learning

With technology-delivered mathematics instruction, students work through content at their own pace during some or all of classroom time, with an instructor providing individualized attention, delivery of instruction, and technology support. Computer-based learning includes course redesign models, in which some or all of face-to-face instruction is replaced with self-paced online curriculum modules (Epper & Baker, 2009; Twigg, 2005); hybrid or blended online learning; and forms of computer-based learning where the traditional course structure is maintained and the instructor still has a role in the classroom.

None of the studies of computer-based learning employed a rigorous research design. Lovett, Meyer, and Thille (2008) randomized students in their study on the impact of Carnegie Mellon University's Open Learning Initiative (OLI) statistics course, but because the treatment condition is an eight-week OLI-Statistics course and the comparison condition is a 15-week traditional course, it is impossible to disentangle the effects of the OLI pedagogy from the possible effects of acceleration. The highest quality study used a quasi-experimental design and found that college algebra students in course sections using ALEKS (Assessment and Learning in Knowledge Spaces), a computer-adaptive online assessment and learning program, experienced small gains in math learning compared to students in traditional algebra course sections (Hagerty & Smith, 2005). However, the final results do not account for students who enrolled late or withdrew from the course; differential attrition could have biased the results.

Research Recommendations

All of the pedagogical practices discussed above may have the potential to improve the outcomes of developmental math students, but given the poor internal validity of many of the studies, it is difficult to infer whether most of the pedagogy is effective in practice. The following are recommendations for future research on developmental math instruction.

Methodological Recommendations

Since it is difficult to conduct randomized experiments in school settings, researchers should collect information on student abilities and demographics and control for observable differences between groups using statistical methods. This would provide more convincing evidence that differences in outcomes are due to the instructional intervention rather than preexisting differences between treatment and control students. If the treatment takes place across multiple classrooms, participating instructors should be similar along observable measures and teach both a

treatment section and a control section. This would help ensure that the effects of an instructional practice can be disentangled from the impact of individual instructors.

In higher education research, it is important to consider that course scheduling can influence the types of students that register for each course, and the characteristics on which these students differ may be related to educational performance. Offering treatment and control sections at similar times reduces the likelihood that outcomes are influenced by student characteristics related to the time of day the course sections are offered.

Multiple-choice math tests and other standardized assessments are often used to study the effects of reform-based math instructional practices, but traditional assessments may not measure the skills that reform-based pedagogy promotes (Hamilton et al., 2003). However, assessments designed specifically for a study may not be fair outcome measures if they only assess the skills taught in the treatment classrooms. In assessing innovations in pedagogy, a combination of standardized and alternative measures may be most appropriate.

Directions for Future Research

Conducting rigorous evaluations of computer-based learning in developmental math is essential to furthering our understanding of how it affects student outcomes. Since redesigning courses with computer-based instruction involves substantial changes to the way course content is delivered, outcomes may be due to any number of changes, including when students can access course content and the pedagogy utilized in each model. There is variation in how much time students spend working through the course material on their own, and some models emphasize individualized attention from instructors and small-group work more than others (Twigg, 2003, 2005). Further, some computer-based course content may allow for investigative problem solving or discovery-based learning, while other software programs rely on drill-and-practice problems. All of these components of course redesign may impact students in different ways, and they may even have differential effects for different types of students. Future research should aim to isolate the effects of different components of course redesigns or to assess the possible differential effects of this reform on student subgroups.

Unlike the studies in the application and computer-based learning sets, which typically take place in colleges and universities, the studies in the other sets target diverse student populations. Overall, the evidence suggests that interventions result in slightly larger effect sizes for students with learning disabilities and lower-achieving students than for typical students. Future research may provide a more focused examination of how innovative pedagogy could promote the foundational skills that adult students need to move beyond basic math.

A final priority for developmental education research is designing and investigating the impact of more balanced instructional approaches that promote all strands of mathematical learning. This is especially important since the Mathematical Association of America and American Mathematical Association of Two-Year Colleges recommend replacing traditional college algebra courses with modeling-based college algebra courses, in which students solve problems situated in real-world contexts by creating and interpreting mathematical models (Katz, 2007). Studies suggest that application-oriented instructional approaches may support some strands of mathematical proficiency but

do not improve procedural fluency. A challenge for researchers and practitioners is to develop modeling-based approaches that improve students' math understanding as well as their performance on traditional standardized tests of mathematics achievement.

Instructional Recommendations

A number of rigorous studies of student collaboration and problem representation showed positive results. Adaptation and evaluation of these pedagogical practices ought to be considered for the developmental mathematics classroom.

Structured Student Collaboration

Many instructors may use cooperative learning in informal ways, but research suggests that cooperative learning may not be effective unless it is systematically integrated into a course. Rigorous studies of student collaboration found that students benefited from cooperative learning methods in which all students played a role in working toward a shared goal.

Applications of structured student collaboration in developmental math include collaborative problem-solving activities that have a group grade tied to them. In Dees's (1991) study, students received partial instructions to a math problem and worked in groups to understand the instructions and solve it. One group member was randomly chosen to explain the solution. Since the group's grade was based on this explanation, students had to collaborate to ensure everyone in the group understood the solution steps and final answer. Activities like this could be used to supplement more traditional instructional practices in the developmental math classroom.

Improving Problem Representation

No developmental education studies focused on problem representation, but rigorous studies involving other student populations demonstrate how instructors can improve student learning outcomes by integrating problem representation instruction into their lectures and interactive board work. In a study by Brenner et al. (1997), pre-algebra students learned to represent problems using graphs, diagrams, tables, pictures, and equations and to solve problems using multiple representations. Similarly, in a study by Chappell (2006), faculty represented calculus concepts numerically, algebraically, and graphically in their lectures, and students were expected to solve problems using these representations on homework and assessments. Evidence from both of these studies suggests that improving students' problem representation skills may improve math learning outcomes.

Developmental math instructors may want to consider modeling problems numerically, algebraically, and graphically in their lessons and having students represent and solve problems in multiple ways on homework and assessments. This would require more time for lesson preparation, a change in the content of homework and assessments, and lessons that spend more time on each concept, which may reduce the time spent working on procedural fluency. It would be useful for researchers to design and evaluate balanced instructional approaches that teach all strands of mathematical proficiency, but individual instructors could also experiment with ways to focus on problem representation while still providing students with practice on traditional equation solving.

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